Crack Propagation Modeling Using the Extended Isogeometric Analysis Technique

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Abstract. In this work, we implemented the extended isogeometric method for cracked structures in 2D. In order to approximate, the displacement fields we have worked with interpolation functions that are based on Non-Uniform Rational B-spline (NURBS). The traditional approximations used in the isogeometric method are extended by the insertion of enrichment functions, which are capable of capturing discontinuities and singularities in the crack tip. This method allows modeling the cracks without being in conformity with a given mesh. For the purpose of showing the ability to model cracks by this method, the result obtained by the extended isogeometric method is compared with the XFEM method.

Keywords: XIGA · XFEM · CRACK · B-spline

1 Introduction

Nowadays, the failures of many engineering structures caused mainly from crack-like surface defects. For this reason, the evaluation of different failure modes includes cracking is of major importance to ensure the exploitation of these structures. As a result, the treatment and prediction of cracks is a challenge for scientific researchers and finite element specialists. Even if the various works are done by the MEF [1] for the calculation of structures, it remains unable to simulate the problems having discontinuities caused by cracks, holes, other bi-material interfaces and modeling the propagation of discrete cracks. This last requires a remeshing on each increment and a mesh compliant, which is difficult and very expensive in terms of time.

To solve the numerical difficulties corresponds to the problems produced by the cracks, several techniques were introduced such as, The element free Galerkin approach [2], Boundary element method [3], Extended Finite element method [4], peridynamic models [5], modeling with phase-field [6]. Among all the various processing techniques

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for modeling structures, the most used method so far is the XFEM, this method based on the unit partition [7], and it allowed meshing the structure without taking into account the crack to describe the opening of the crack and the singularity at its tip, special shapes functions are inserted. The approximation of the geometry and the solution performed by various basic functions during the use of such calculation technique as FEM, XFEM. Consequently, they present discretization errors [8]. To eliminate these errors, Hugues et al. [9] developed a new computation tool called Iso- Geometric Analysis (IGA). This method created a relationship between computer-aided design CAD and finite element method FEM. The idea is to apply the basic functions of CAD in the geometric representation and to build the finite approximations. While the basic Lagrangian function with the finite element method is the best known in the CAE, the most basic functions widespread use of CAD are non-uniform and rational B-spline functions (NURBS). The use of the IGA approach offers us the ability to make a simple refinement as well as an exact description of the engineering structures, so the accuracy and robustness make distinguish the method from the conventional method.

In recent years the IGA has appeared successfully in various engineering problems [10] particularly in the mechanics of fracture [11]. Concerning cracking problems, to describe the phenomenon of discontinuity on the long lips of the crack and the singularity at the crack tip, the enrichment functions through the unit partition method integrated into IGA. There are many works like De Luycker et al. [12]; he applied the method XFEM and IGA to study the mechanics of linear rupture. In the following Gorashiet et al. [13] was extended this method to an enriched method (under the XIGA name) to capture the behavior of cracked structures in 2D. XIGA has been a real success in the modeling of fracture mechanics [14], cracking of a thin shell [15].

This work is done to study a stationary crack in a 2D structure using the implementation of the XIGA. We will present the concept of the XFEM and a generalization on the isogeometric method then a description of the XIGA method. Finally, simple examples to compare the new approach with the XFEM approach.

2 The Extended Finite Element Method

2.1 Representation of the Crack

The XFEM method has appeared as an alternative to the classical finite element method, it based on the principle of unit partition [7], it is able to follow the crack without being in conformity with the mesh thus it doesn't require to do this remeshing operation.

At first, the shape of the crack meshed with triangles independently of the structural mesh.

Second, the level set functions called (ψ, φ) are calculated at nodes of the geometry around to the crack surface as despite in Fig. 1.

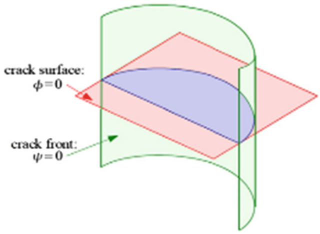


Fig. 1. Level set function

2.2 Enrichment Approach

The essential idea of XFEM is to add enrichment function to the classical finite element method [16]. Therefore, after the crack is presented by level set function, it is achievable to enrich the displacement field by inserting enrichment shape functions and related degrees of freedom to the discretization. In fact, the mesh of the structure is created without any regard of the crack and XFEM could take it into account only by appending specific enrichment function. According to the partition of unity, discontinuous shape function (called Heaviside) presented by Eq. (1) is added at the nodes corresponds to the elements completely cut by the crack, thus the singular shape functions described by Eq. (2) are added at the nodes of the elements including the crack front.

$$\mathrm{H(x)} = \begin{cases} +1 \text{ if } \varphi(\mathrm{x}) > 0 \text{ above the crack} \\ -1 \text{ if } \varphi(\mathrm{x}) < 0 \text{ Below the crack} \end{cases} \quad (1)$$

$$\begin{aligned} F_1(\mathrm{x}) &= \sqrt{r}\sin(\theta/2),\ F_2(\mathrm{x}) = \sqrt{r}\sin(\theta/2)\sin(\theta), \\ F_3(\mathrm{x}) &= \sqrt{r}\cos(\theta/2),\ F_4(\mathrm{x}) = \sqrt{r}\cos(\theta/2)\sin(\theta). \end{aligned} \quad (2)$$

With r and θ are polar coordinates attached to the crack tip.

Let Ω represent the solution domain, Ω_H the nodes of the elements cut by the crack and Ω_F the nodes of the elements which crack tip placed. So the displacement field with the XFEM discretization can be represented as follows:

$$U(x) = \sum\nolimits_{i\in\Omega} N_i(x)u_i + \sum\nolimits_{i\in\Omega_H} N_i(x)H(x)a_i + \sum\nolimits_{i\in\Omega_F} N_i(x)\left[\sum\nolimits_{j=1,..4} F_J(x)b_{J,i}\right] \quad (3)$$

Where N_i is the finite element shape function linked to the node i, then u_i, a_i, $b_{J,i}$ represent the classical, discontinuous and singular degrees of freedom [16].

The isotropic enrichment functions represented in Eq. (2) are the basic and the most used type of enrichment functions [17]. Since the structure has an elastic behavior, the use of this type of enrichment function increases the precision of the approximation.

In order to obtain an optimal discretization, that is to say, to have a minimum number of degrees of freedom, Fig. 2 Shown the topology of enrichment followed.

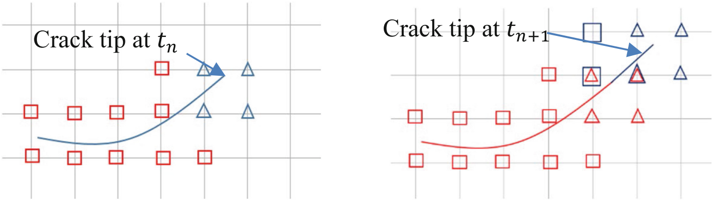


Fig. 2. Enrichment technique

Only the elements that have cut through the crack enriched. There are two strategies of enrichment; geometry enrichment which based on the implementation of a fixed zone around the crack tip. This method improves the convergence of the solution [18], for the other approach that might have been examined is the degrees of freedom, collecting that has been presented in [19].

During the propagation of the crack, novel singular and discontinuous degrees of freedom are included in the approximation (see Fig. 2).

3 A Brief Presentation of the Isogeometric Analysis IGA

The most used basic functions of IGA are the NURBS; they commonly used to approximate the fields of displacement and geometry in a given domain. IGA has made it possible to eliminate all discretization errors caused by the geometric approximation.

The vector of knots ξ can be ordered in the following way;

$$\xi = \{\xi_1, \xi_2, \xi_3, \ldots\ldots\xi_{n+p+1}\} \tag{4}$$

With $\xi_i \in \Re$ is the i-th knot, i is the index of the node, i ∈ {1,2,3……n + p + 1} with p is the polynomial degree of the b-spline and n is the number of the associated function.

3.1 Shape Function

For a given order p, the B-spline basis functions are defined recursively from the node vector by applying the cox-of-boor formula, starting with the constant functions (P = 0):

$$N_{i,0} = \begin{cases} 1 & \xi_i \leq \xi < \xi_{i+1} \\ 0 & \text{Otherwise} \end{cases} \tag{5}$$

Then, we build for p > 0:

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi) \tag{6}$$

A curve NURBS of order p defined by n + 1 control point:

$$\mathrm{P}(\xi) = \sum_{i=0}^{n} R_{i,p}(\xi) X_i \tag{7}$$

Where:

$$R_{i,p} = \frac{N_{i,p}(\xi) w_i}{\sum_{i=0}^{n} N_{i,p}(\xi) w_i} \tag{8}$$

We take note that:
$\{R_{i,p}\}$: Function NURBS
$\{X_i\}$: $\{X_{i_1}, X_{i_2}\}$ the coordinates of the control point set.

3.2 The Discretization of a Cracked Structure with IGA

In the context of elastic linear mechanics, consider a domain with the following boundary conditions (see Fig. 3):

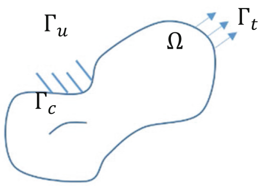


Fig. 3. Mechanical problem

Γ_u: Dirichlet boundary, Γ_t: Neumann boundary, Γ_c: crack surface

Therefore, the equilibrium equation and the boundary conditions of this problem can be represented in the following form:

$$\nabla.\sigma + b = 0 \text{ on } \Omega \tag{9}$$

$$\sigma \cdot \mathrm{n} = \bar{\mathrm{t}} \text{ on } \Gamma_t \tag{10}$$

$$\sigma \cdot \mathrm{n} = 0 \text{ on } \Gamma_c \tag{11}$$

$$\mathrm{u} = \bar{u} \text{ on } \Gamma_u \tag{12}$$

Where, σ, b, u corresponds respectively to Cauchy stress tensor, body force, displacement.

The law hook gives the behavior law of a linear elastic problem:

$$\sigma = D\,\varepsilon \tag{13}$$

With the deformation, a tensor can be written in the form:

$$\varepsilon = \nabla_s \mathrm{u} \text{ on } \Omega \text{ with } \nabla_s = \begin{bmatrix} \partial/\partial x & 0 \\ 0 & \partial/\partial y \\ \partial/\partial y & \partial/\partial x \end{bmatrix} \tag{14}$$

And D represents the elastic matrix.

Any deformable-body under external stress has the weak formulation of the following equilibrium equation:

$$\int_{\Omega} \sigma : \mathrm{u}\, d\Omega - \int_{\Omega} \mathrm{b} : \mathrm{u}\, d\Omega - \int_{\Gamma_t} \bar{t} : \mathrm{u}\, d\Gamma_t = 0 \tag{15}$$

Through this equation, we can obtain the following discrete equation system:

$$[\mathrm{K}]\{\mathrm{U}\} = \{\mathrm{f}\} \tag{16}$$

With, K represents the global stiffness matrix.

U the vector of nodal unknowns

f the vector of forces

In this work, the displacement field approximation and domain geometry are done by integrating the NURBS of the IGA technique.

$$U^h(\xi) = \sum_{i=1}^{n_{en}} R_i(\xi) u_i \tag{17}$$

$$X(\xi) = \sum_{i=1}^{n_{en}} R_i(\xi) X_i \tag{18}$$

Hence, R (ξ) represents the basic function NURBS and n_{en} = (p + 1) × (q + 1) defines the number of control points in ξ_1 and ξ_2 directions, then p and q are the orders of the curve in the direction ξ_1 and ξ_2 respectively.

ξ = (ξ_1, ξ_2) represents the parametric space

X = (X_1, X_2) Represents physical coordinates

U^h Represents the displacement approximation

4 Extended Isogeometric Analysis (X-IGA)

This new tool (XIGA) has been improved to allow the modeling of cracks without taking into account the mesh chosen for the calculation. In this method, using the unit partitioning principle in enriching the conventional displacement approximations by suitable enrichment functions. To model the discontinuities, various types of enrichment exist. XIGA removes all unchangeable compatible mesh. To extract local discontinuous fields and singular fields using the concept of the XFEM method, the formula for the enriched displacement approximation is represented as follows:

$$U^h(\xi) = \sum\nolimits_{i=1}^{n_{en}} R_i(\xi)u + \sum\nolimits_{j=1}^{n_s} R_i(\xi)\{H(\xi) - H(\xi_j)\}a_j + \sum\nolimits_{k=1}^{n_t} R_k(\xi) \sum\nolimits_{\alpha=1}^{4} [\beta_\alpha(\xi) - \beta_\alpha \xi_k)]b_k^\alpha \quad (19)$$

In this formulation above, the polynomials of Lagrange have been replaced by basic functions NURBS R_i to have a new numerical method. u_i represents the traditional degree of freedom and a_j are the enriched degree of freedom corresponds to the crack lip, b_k^α introducing the enriched degree of freedom corresponds to the crack tip. n_s and n_t respectively represent the number of basis functions having the crack lips in their support and the number of basic functions corresponds to the crack tip in the support as shown in Fig. 4.

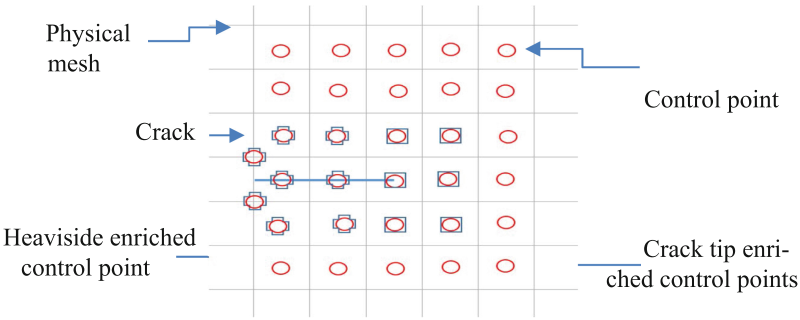


Fig. 4. A quadrature NURBS mesh with the enrichment approach

In the formula below, H(ξ) represents the Heaviside function [16] it receives the value +1 when it is in the upper side of the crack and −1 on the opposite side. $\beta_\alpha(\xi)$ represents the functions of enrichment of the crack tip so the Eq. (20) represents these functions in local polar coordinates (r,θ):

$$\beta_\alpha(\xi) = \sqrt{r}\,(\sin(\theta/2), \sin(\theta/2)\sin(\theta), \cos(\theta/2)\cos(\theta)) \quad (20)$$

The various matrices constituting the XIGA model can be represented in the following way:

$$[K_{ij}] = \begin{bmatrix} K_{ij}^{uu} & K_{ij}^{ua} & K_{ij}^{ub} \\ K_{ij}^{au} & K_{ij}^{aa} & K_{ij}^{ab} \\ K_{ij}^{bu} & K_{ij}^{ba} & K_{ij}^{bb} \end{bmatrix} \quad (21)$$

$$\{\mathrm{f}\} = \left\{f_i^u, f_j^a, f_k^{b1}, f_k^{b2}, f_k^{b3}, f_k^{b4}\right\}^T \quad (22)$$

$$f_i^u = \int_\Omega R_i^T \cdot b\, d\Omega + \int_\Gamma R_i^T \cdot \bar{t}\, d\Gamma \tag{23}$$

$$f_j^a = \int_\Omega R_j^T \{\mathrm{H}(\xi) - \mathrm{H}(\xi_j)\}\ \mathrm{b}\, d\Omega + \int_\Gamma R_j^T \{\mathrm{H}(\xi) - \mathrm{H}(\xi_j)\}\ \mathrm{t}\, d\Gamma \tag{24}$$

$$f_k^{b\alpha} = \int_\Omega R_k^T \{\beta_\alpha(\xi) - \beta_\alpha(\xi_k)\}\, \mathrm{b}\, d\Omega + \int_\Gamma R_k^T \{\beta_\alpha(\xi) - \beta_\alpha(\xi_k)\}\, \mathrm{t}\, d\Gamma \tag{25}$$

$$K_{ij}^{r,s} = \int_\Omega B_i^{rT} = \mathrm{D} B_j^s d\Omega \text{ with } \ \mathrm{r, s = u, a, b} \tag{26}$$

$$B_i^a = \begin{bmatrix} (R_i)_{,x1} H & 0 \\ 0 & (R_i)_{,x2} H \\ (R_i)_{,x2} H & (R_i)_{,x1} H \end{bmatrix} \tag{27}$$

$$B_i^{b_\alpha} = \begin{bmatrix} (R_i \beta_\alpha)_{,x1} & 0 \\ 0 & (R_i \beta_\alpha)_{,x2} \\ (R_i \beta_\alpha)_{,x2} H & (R_i \beta_\alpha)_{,x1} \end{bmatrix} \ (\alpha = 1, 2, 3, 4) \tag{28}$$

$$B_i^b = [B_i^{b1} B_i^{b2} B_i^{b3} B_i^{b4}] \tag{29}$$

5 Numerical Result

To evaluate the performance and accuracy of the XIGA, we will take a simple problem of elastic linear mechanics; an edge and center crack problems. We used the simulation code developed by [20] and the result of the simulation will be compared with the XFEM method. We have chosen an order 3 for basic functions NURBS in both parametric directions. To do the simulation we took the properties of the following material: E = 10^7 MPA, ν = 0,3 for edge crack and E = 3 10^7 MPa, ν = 0,3 for center crack. We assumed the plane strain condition.

5.1 Example of a Plate with an Edge Crack

The geometry of the plate is of dimension W × L as the Fig. 5 indicate. In the top edge, uniform stress is applied σ = 1. a is the crack length with a = 0,45. In the bottom right corner of the domain U1 = U2 = UR3 = 0 and in the top right corner U1 = UR3 = 0. The result of the displacement Uy for an edge crack is represented in the Fig. 6.

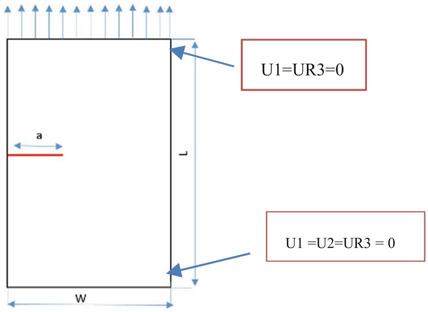
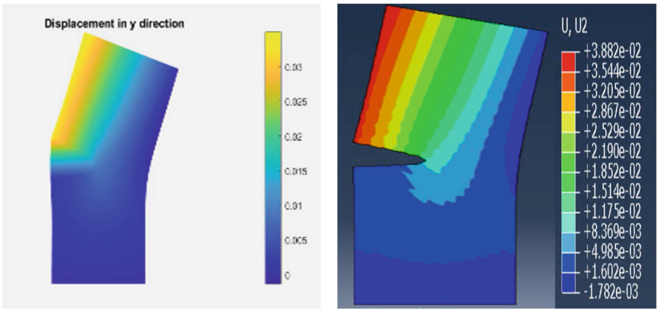


Fig. 5. Edge crack in tension



(a) Illustration with XIGA (b) illustration with XFEM

Fig. 6. Displacement contour plot Uy for an edge crack

5.2 Example of a Plate with a Center Crack

The geometry of the plate is of dimension W × L as the Fig. 7 indicate. In the top edge, uniform stress is applied $\sigma = 1$. a is the crack length with a = 0,25. The result of the displacement Uy for a center crack is represented in the Fig. 8.

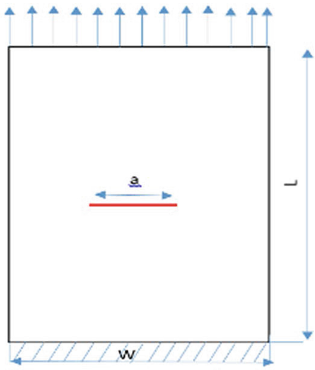
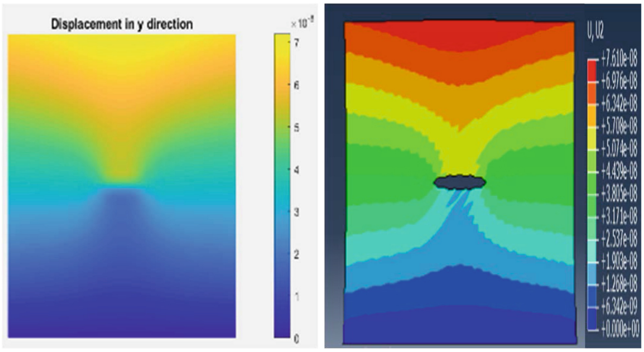


Fig. 7. Plate with a center crack



(b) Illustration with XIGA (b) illustration with XFEM

Fig. 8. Displacement contour plot Uy for a center crack

The results obtained by the XFEM method and the XIGA method are identical so we can use this novel method to study the various defects produced by the crack and to follow the crack propagation.

6 Conclusion

In this work, the simulation of a cracked structure is done by the implementation of the extended isogeometric analysis X-IGA method, which is the extended of the isogeometric method. By applying the concept of the XFEM method, the enrichment functions are integrated to extract the discontinuous and singular local fields. The integration of this method in the finite element calculation makes it possible to make a connection between the CAD and FEM to minimize the errors caused by the discretization thus obtaining the results more efficient. For future researches, we decide to apply this method to follow the propagations of the cracks on the shell structures, which are subjected to internal pressures like pipelines.

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